Rotationally Invariant 3D Shape Contexts Using Asymmetry Patterns

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3D Geometric Descriptors

- We look for local properties that allow distinguishing a given point from the rest of points of the mesh
3D Geometric Descriptors

- **Distance-based**
  - Spin Images (SI)
  - 3D Shape Contexts (3DSC)
  - Unique Shape Contexts (USC)

- **Orientations-based**
  - Point Feature Histograms (PFH)
  - Fast Point Feature Histograms (FPFH)
  - Signature of Histograms of Orientations (SHOT)

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3D Shape Contexts (3DSC)

- Histogram of distances on a spherical support region
- Logarithmic spacing of radial bins
  - Bins closer to the interest point are smaller to capture detail
  - More distant bins are bigger, to capture "context"
- Volumetric normalization
  - To compensate for sampling density
  - To compensate for bin size

3DSC Orientation and Matching

- 3DSC (Frome et al. 2004)
  - Test all possible rotations and pick the best match
- Unique Shape Contexts (Tombari et al. 2010)
  - Resolve the azimuth origin by SVD + sign disambiguation
  - Lower accuracy than 3DSC
- Harmonic Shape Contexts (Frome et al. 2004)
  - Use spherical harmonics and keep just the module
  - High computational cost
Asymmetry Patterns Shape Contexts (APSC)

- We define a new family of descriptors by extracting asymmetry patterns from 3DSC
- APSC properties:
  - Rotationally invariant by construction
  - Faster comparison (matching)
  - Small increase in computational cost w.r.t. 3DSC (typically 5% to 10%)
  - Can be defined from a variety of patterns
    - Useful when targeting different types of points
- We can get several APSC descriptors for the price of one (with a small overhead).
- Experiments on 3D facial landmark localization

Measuring symmetry

- We start from a measure of approximate rotational symmetry in 2D, presented by Guo et al (2010):

\[
S_c(m, \phi) = \frac{\text{Area}(m \cap R(m, \phi))}{\text{Area}(m)}
\]

- \(S_c(m, \phi)\): continuous degree of rotational symmetry of shape \(m\) at a rotation \(\phi\)
- \(R(m, \phi)\): Rotation of shape \(m\) by an angle \(\phi\)

Measuring symmetry

\[ S_e(m, \phi) - \frac{\text{Area}(m \cap R(m, \phi))}{\text{Area}(m)} \]

No need to set the origin for the rotation!

Adapting symmetry to 3DSC

- Consider the bins at fix elevation and radius
  - We get a stripe that varies with Azimuth
  - The bin values are a sequence \( m \) of \( N_A \) values (the number of Azimuth bins)
    \[ m_j = x_{i,j,k}, \quad m_j \geq 0 \forall j \in [1; N_A] \]
  - \( x_{i,j,k} \) are the bins of a 3DSC
    - \( i = \) Elevation
    - \( j = \) Azimuth
    - \( k = \) Radius
Adapting symmetry to 3DSC

- We can redefine symmetry as:
  \[ S(m, a) = \frac{\sum_j \min(m_j, m_{j+a})}{\sum_j m_j} \]
  - Rotation is replaced by a discrete shift by \( \alpha \) units
  - Additions \( j + \alpha \) are operations modulo \( N_A \)
- The formula for its complement (asymmetry) becomes:
  \[ A(m, a) = 1 - S(m, a) = \frac{1}{2} \sum_j |m_j - m_{j+a}| \]
- We can drop the ring-constant elements to get:
  \[ A_1(m, a) = \sum_j |m_j - m_{j+a}| \]

Asymmetry patterns

- The difference of absolute values of a ring and a shifted version of itself measures its asymmetry at that shift
  \[ A_1(m, a) = \sum_j |m_j - m_{j+a}| \]
- We can define an asymmetry pattern as follows:
  \[ P_A(m) = A_1(m, 1), A_1(m, 2), \ldots, A_1(m, \lfloor \frac{N_A}{2} \rfloor) \]

No need to set the origin for the rotation!
World-Map example

Example of 3DSC descriptor

APSC descriptor

Side effects of rotational invariance

Example of 3DSC descriptor

APSC descriptor

The APSC descriptor remains unchanged
Side effects of rotational invariance

Example of 3DSC descriptor

APSC descriptor

The APSC descriptor remains unchanged

Side effects of rotational invariance

Example of 3DSC descriptor

APSC descriptor

The APSC descriptor remains unchanged
Selecting the spatial patterns (1-ring)

Example of 3DSC descriptor \( (x_{i,j,k}) \)

Spatial Pattern:

A: \[ m_j = x_{i,j,k} \]

The disconnection is not resolved but the spatial relations captured are different

2-ring patterns

Spatial Pattern:

A+E: \[ m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i+1,j,k} \]

\[ A_2(m, a) = \sum_j |m_{1,j} - m_{1,j+a}| + |m_{2,j} - m_{2,j+a}| \]

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Sequence(s) equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_AER</td>
<td>( m_j = x_{i,j,k+j} )</td>
<td>Azimuth-Radius diagonal</td>
</tr>
<tr>
<td>D_AER</td>
<td>( m_j = x_{i+1,j,k+j} )</td>
<td>Azimuth-Elevation-Radius diagonal</td>
</tr>
<tr>
<td>A+E</td>
<td>( m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i+1,j,k} )</td>
<td>Azimuth ring + Elevation neighbors</td>
</tr>
<tr>
<td>A+R</td>
<td>( m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i,j,k+1} )</td>
<td>Azimuth ring + Radial neighbors</td>
</tr>
<tr>
<td>A+D_AER</td>
<td>( m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i+1,j,k+1} )</td>
<td>Azimuth ring + Azim-Elev-Rad diagonal</td>
</tr>
</tbody>
</table>
Evaluation of local accuracy

Example of similarity maps computed using an average descriptor

Similarity maps are typically not as good as the one for the nose tip

We restrict the analysis to a local neighborhood of the target

Definition: Expected Local Accuracy

- Is the expected distance from the vertex obtaining the maximum score to the ground truth position, but only searching on a neighbourhood of radius $r$
Expected Local Accuracy

- Is the expected distance from the vertex obtaining the maximum score to the ground truth position, but only searching on a neighbourhood of radius $r$.
Results (144 facial scans, 6-fold cross validation)

<table>
<thead>
<tr>
<th>Link</th>
<th>HSC</th>
<th>USC</th>
<th>3DSC</th>
<th>APSC (DAR)</th>
<th>APSC (DAER)</th>
<th>APSC (A+R)</th>
<th>APSC (A+DAR)</th>
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<tbody>
<tr>
<td>ex</td>
<td>1.3 (2-24)</td>
<td>1.9 (3-25)</td>
<td>1.4 (3-2)</td>
<td>1.3 (3-2)</td>
<td>1.3 (3-2)</td>
<td>1.3 (3-2)</td>
<td>1.3 (3-2)</td>
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<tr>
<td>m</td>
<td>4.5 (16-90)</td>
<td>n.a.</td>
<td>4.3 (13-88)</td>
<td>3.9 (19-88)</td>
<td>5.4 (13-88)</td>
<td>4.7 (14-89)</td>
<td>3.1 (8-88)</td>
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<tr>
<td>a</td>
<td>1.8 (3-200)</td>
<td>n.a.</td>
<td>1.7 (3-200)</td>
<td>1.6 (4-64)</td>
<td>2.3 (4-200)</td>
<td>2.0 (4-200)</td>
<td>1.7 (4-200)</td>
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<tr>
<td>ac</td>
<td>2.1 (5-28)</td>
<td>5.8 (14-25)</td>
<td>4.7 (7-25)</td>
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<td>at</td>
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<td>12.2 (14-200)</td>
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Global performance

Average over all 22 landmarks

![Graph showing average local accuracy vs. search radius]
Computational complexity

- Tests on triangulated meshes averaging 21K vertices and 1.5mm mesh resolution
- Intel Xeon E5320 @1.86 GHz
- Time values normalized (3DSC time = 1.0)

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Computation</th>
<th>Matching</th>
<th>≈ 0.083</th>
<th>≈ 0.083</th>
<th>≈ 0.042</th>
<th>≈ 0.042</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSC</td>
<td>11.1</td>
<td>( \frac{N_{SH}(N_{SH}+1)}{2N_FN_A^2} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USC</td>
<td>1.23</td>
<td>( \frac{1}{N_A} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APSC (D_{AR})</td>
<td>1.05</td>
<td>( \frac{1}{2N_A} )</td>
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<tr>
<td>APSC (D_{AER})</td>
<td>1.09</td>
<td>( \frac{1}{2N_A} )</td>
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<tr>
<td>APSC (A+E)</td>
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<td>APSC (A+R)</td>
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APSC computation

- Once an APSC descriptor is built
  - Additional APSC can be built at little extra cost
  - We also get a 3DSC

Common building time

\( +5\% \)

\( +9\% \)
Conclusions

- **Individual APSC descriptors:**
  - Rotationally invariant, globally comparable to 3DSC
  - Are 5% - 10% more expensive to build than 3DSC
  - Are much faster to evaluate than 3DSC and require half its memory

- **Groups of APSC**
  - Can be computed incrementally with little additional cost for each additional descriptor
    - The extra cost is only to build the descriptor, not to match it
  - We also get the 3DSC descriptor, in case there are no APSC with sufficient accuracy
  - Under similar performance APSC are much faster to evaluate
Craniofacial dysmorphology

- Craniofacial geometry has been suggested as an index of early brain dysmorphogenesis in neuropsychiatric disorders
  - Down syndrome
  - Autism
  - Schizophrenia
  - Bipolar disorder
  - Fetal alcohol syndrome
  - Velocardiofacial syndrome
  - Cornelia de Large syndrome
  - Brandet-Biedl syndrome
  - ...

The Face3D project

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The partners in the project are:
- The University of Glasgow
- Royal College of Surgeons in Ireland
- Dublin City University
- Institute of Technology, Tralee
- University of Limerick

THANK YOU FOR YOUR ATTENTION

http://fsukno.atspace.eu