

Automated packing systems: Review of industrial implementations.

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ABSTRACT

The problems involved in the automated packing and nesting of irregular shapes are not only of theoretical importance, but have considerable industrial interest. The ability to manipulate objects under visual control is one of the key tasks in the successful implementation of robotic, automated assembly and adaptive material handling systems. Such systems must be capable of dealing with a wide range of variable products and of operating in industrial environments that are not overly constrained. These systems will need to include the ability to manipulate arbitrary shapes in a flexible manner. Therefore, to automate this part of the manufacturing process we need to develop automated material handling systems that combine machine vision techniques and flexible packing strategies.

A rich theoretical background to the problems that occur in the automation of material handling can be found in operations research, production engineering, systems engineering and automation, more specifically machine vision, literature. This work has contributed towards the design of intelligent handling systems. This paper will review the application of these automated material handling and packing techniques to industrial problems. The discussion will also highlight the systems integration issues involved in these applications.

An outline of one such industrial application, the automated placement of shape templates on to leather hides, is also discussed. The purpose of this system is to arrange shape templates on a leather hide in an efficient manner, so as to minimize the leather waste, before they are automatically cut from the hide. These pieces are used in the furniture and car manufacturing industries for the upholstery of high quality leather chairs and car seats. Currently this type of operation is semi-automated. The paper will outline the problems involved in the full automation of such a procedure.

1. INTRODUCTION

Early research into determining optimal packing/nesting configurations can be traced back to Johannes Kepler in 1611 when he tried to determine if the most efficient method of packing identical spheres was an arrangement now known as face-centred cubic lattice. This consists of placing a bottom layer of spheres in a bounded region. Each successive layer is then arranged so that the spheres occupy the gaps of the layer below (Stewart 1991; 1992). This is the arrangement greengrocers use to stack oranges. Although the stacking of oranges in this way seems intuitive, researchers are still unable to prove that this stacking configuration is the most efficient.

2. THE PACKING PROBLEM - AN OUTLINE OF CURRENT RESEARCH.

The field of operations research has been active in developing techniques for the automated packing/nesting of two and three-dimensional shapes. Dowsland and Dowsland (1992) have recently reviewed the work done in this area. As well as considering the application of operational research techniques to a full range of packing tasks, this review outlines the future trends in packing research from this perspective. Sweeney and Paternoster (1992) also produced a recent review of the packing problem. The authors have grouped the publications according to the three main solution methodologies, these are summarized below:

- Sequential assignment heuristics, packing of patterns based on a set of assignment rules. The majority of heuristic approaches consist of determining what order and orientation the pieces should be packed in.
- Single-pattern generating procedures such as dynamic programming based algorithms which try and reuse a single 'optimal' packing configuration. For example, in the two-dimensional rectangular packing problem the solution is built up by considering partial solutions within smaller containing rectangles (Dowsland and Dowsland 1992).
- Multi-pattern generating procedures such as linear programming¹ based approaches which consider the interaction between patterns. This approach requires the solutions to be rounded and are, therefore, also heuristic in nature. The packing task can be formulated as a binary integer problem in which a single variable represents each possible piece position. For example a rectangle could be represented by a top left coordinate. A major concern with this approach is the production of a physical design from the values of the variables in the integer programming solution (Dowsland and Dowsland 1992).

A practical review of two and three-dimensional packing issues and solution methods can be found in Dowslands (1985) paper. The majority of the applications outlined in this review are based on two-dimensional packing techniques. Many of the three-dimensional problems are tackled by applying two-dimensional techniques on a layer by layer basis. Most published work in the area of three-dimensional packing is limited due to its complexity, and the applications that are discussed tend to be concerned with the loading of shipping containers. The paper also summarises some of the practical requirements in pallet loading, these include:

- The stability of the loading stack.
- The load bearing ability of the items in the stack.
- Ease of stacking.
- Air circulation requirements of certain products in a stack.

Carpenter and Dowsland (1985) expand on these system considerations. Dowsland (1985) also reviews some of the heuristic approaches used for packing a given set of identical, and non-identical, rectangular items into a containing rectangle. This extensive review covers the key areas in automated packing, such as optimality versus efficiency and the measurement of a packing systems performance. The basic conclusion of the author is that although some very high packing densities have been reported in the literature, as yet there is no generic heuristic approach that can be applied to the two-dimensional packing task. Solutions reported tend to be very application specific.

¹ This is a technique that is used to provide "a mathematical description (or model) of a real-life problem in which something needs to be maximised (e.g. profits or security) or minimized (e.g. costs or risks)" (Devlin 1988). Optimization is achieved by the suitable choice of a number of parameter values. This strategy makes much stronger demands on the program structure when compared to heuristic techniques.

2.1 Packing of regular shapes.

The main emphasis of the early research into packing tended to concentrate on the well constrained problem of packing regular shapes. This task usually consists of packing two-dimensional regular shapes into a well defined scene (the term *scene* is used, in this paper, when referring to a region of space into which an arbitrary *shape* is placed), such as a rectangle (Chuang, Garey and Johnson 1982; Brown 1971). The main industrial applications are in the area of pallet packing (Carpenter and Dowsland 1985) and container loading (Bischoff and Marriott 1990). Other applications include efficient VLSI design and automated warehousing (Hall, Shell and Slutzky 1990).

2.2 Packing of irregular shapes.

More recently researchers in the field of engineering and science have begun to concentrate on the issues involved in the packing of irregular shapes. Research in this area is constrained by the demands of a given application, so as to make the task more manageable. Batchelor (1991) outlines a technique for the packing of complex shapes based on the use of the minimum area bounding rectangle.

Qu and Sanders (1987) discuss a heuristic nesting algorithm for irregular parts and the factors affecting trim loss. The application discussed is the cutting of a bill-of-materials from rectangular stock sheets. The author takes a systems approach to the problem and produces some good results. These are discussed in the context of performance measurements which they have developed. While the authors review the published work in this area, they make the important point that although a number of techniques have been developed to enable the flexible packing of irregular shapes, very few of these have been published due to commercial confidentiality. The approach described represents the irregular shapes in terms of a set of non-overlapping rectangles. In fact the authors state that each of the parts in their study can be represented by no more than five non-overlapping orthogonal rectangles. The system places each part in an orientation such that (a) its length > height and (b) the largest complimentary (void) area is in the upper-right corner. The parts are then sorted by non-increasing part height. The shapes are packed into a rectangular bounding region in a raster fashion, building up layers of intermeshed packed shapes. The major disadvantage with this approach are (a) the use of rectangles to approximate the shape to be packed and (b) the assumption that good packing patterns will be orthogonal.

Dori and Ben-Bassat (1984) investigate the nesting of shapes within a polygon rather than a rectangle. The authors discuss the optimal packing of two-dimensional polygons with a view to minimizing waste. The algorithm is only applicable to the nesting of congruent² convex figures. The problem involves cutting a number of similar but irregular pieces from a steel board, this is referred to as the 'template-layout problem'. The authors decompose the task into two sub-problems. The first consists of the optimal (minimal waste) circumscription of the original irregular shape by the most appropriate convex polygon. The remaining problem consists of circumscribing the convex polygon by another polygon that can pave the plane, that is, cover the plane by replications of the same figure without gaps or overlap. This is referred to as the paver polygon. Limitations of this approach include the fact that it is only applicable to congruent convex figures and the assumption that the packing plane is infinite, hence waste in the margin is not considered. Another limitation of this approach is that it can only be applied to convex components with straight sides. Koroupi and Loftus (1991) address the issues raised by Dori and Ben-Bassat (1984), by enclosing the component within a polygon so that the area added is minimal. The identical components, whether regular or irregular, are then nested using paving techniques.

Prasad and Somasundaram (1991) outline a heuristic based computer aided system that will allow the nesting of irregular-shaped sheet-metal blanks. This paper also contains a comprehensive list of the practical constraints one must consider in developing a packing system for sheet metal stamping operations. Constraints such as, blank width, blank separation, grain orientation, and the minimization of scrap. They also highlight the need to align the pressure centre of the blank to be cut out with the axis of the press ram to reduce wear in the guideways of the press. Design requirements, such as

² Figures of the same form and size, differing at most with respect to their orientation and position.

maximizing the strength of the part when subsequent bending is involved, are also considered.

Kothari and Klinkhachorn (1989) present a two-dimensional packing strategy capable of achieving dense packing of convex polygon shapes. The techniques described have been applied to the stock cutting in the hardwood manufacturing industry. This consists of efficiently cutting wooden pieces from a hardwood board so that the pieces are free of defects and aligned in the direction of the grain. This last constraint is needed for strength and aesthetic reasons.

Albano and Sapuppo (1980) discuss a procedure which claims to produce an optimal arrangement of irregular pieces. Manual and semi-automatic approaches to this nesting task are also discussed. The techniques described show how "the optimal allocation of a set of irregular pieces can be transformed into the problem of finding an optimal path through a space of problem states from the initial state to the goal state". The search approach developed makes certain assumptions about the task; (a) the pieces are irregular polygons without holes and (b) that the bounding region is rectangular. Despite this, their paper contains some excellent results. The main applications discussed is that of cloth layout and leather cutting.

Chung, Scott and Hillman (1990) developed an automated nesting system which determines how to cut regular or irregular two-dimensional pieces from a regular or irregular shaped material. One of the application constraints that must be considered is the fact that the material to be cut, an animal hide, contains defective regions. The problem of cutting defective animal hides appears in the leather upholstery and shoe industry, for example. The approach taken implements an objected-orientated representational scheme in conjunction with a heuristic search procedure to determine an efficient nesting position. The authors concentrate on finding an "satisfactory solution" rather than trying to exhaust all possible packing positions for an optimal solution. A solution is deemed "satisfactory" when its yield is better than or equal to the average of a human expert's solution. The overall system has been evaluated by comparing its performance to that of a human expert, and an average yield difference of within 5% has been claimed.

Cuninghame-Green (1989) considers the moving and nesting of irregular shapes within the context of some practical applications such as leather cutting, for the manufacture of shoes, the efficient cutting of boards and the packing of chocolates. In the case of the efficient cutting of the shoe leather patterns, the author outlines some of the applications constraints that occur in practical applications of packing techniques. These include the isotropic constraints of grain matching. For example, if the grain of the material requires that all the shapes to lie in the same orientation, then the resultant layout will waste 46% of the material. Whereas, if the grain allows the shapes to be turned upside down, this figure is reduced to 33%. The interlocking of the shapes can further reduce this figure. As well as developing his own approach based on what he calls "configuration space obstacle or CSO" which shows the possible packing position for any pair of convex polygons in a given orientation, a link is also made between the fundamental ideas behind motion planning (Sharir 1989) to automated packing. To deal with a broader class of shapes, that is shapes other than convex polygons, two approaches are discussed. The first is based on the concept of packing the convex hull of each irregular shape. This has the disadvantage that the shapes cannot be interlocked and as such material can be wasted. The second approach requires the dissection of each irregular shape into convex polygons to which the CSO approach is applied.

More recently Whelan and Batchelor (1991; 1992; 1993) have developed a system that will allow the packing of arbitrary shapes into an arbitrary scene. This packing scheme consists of two major components:

- (a) A *geometric packer (GP)*, based upon the principles of mathematical morphology and which takes an arbitrary shape in a given orientation and puts the shape into place, *in that orientation* (Whelan and Batchelor 1991).
- (b) An *heuristic packer (HP)*, which is concerned with the ordering and alignment of shapes prior to applying them to the geometric packer. This component also deals with other general considerations, such as the conflict in problem constraints and the measurement of packing performance. In addition, it deals with practical constraints, such as the effects of the robot gripper on the packing strategy, packing in the presence of defective regions, and isotropic ("grain" in the material being handled) and pattern matching (Whelan and Batchelor 1992; 1993).

3. PACKING TECHNIQUES IN INDUSTRIAL SYSTEMS.

Lately a growing number of researchers have begun to develop industrial systems that deal with the more difficult robotic tasks (Lee 1989). One such application of this new generation of 'intelligent robotics' is the automated packing (nesting) of parts in an assembly process. Applications of these systems include the automated assembly of small components under visual control (Van der Heijden 1985), air motor assembly (Adept 1991), automated gasket and carburettor mating (Shoureshi *et al* 1989) and automatic shirt collar assembly (Delgrange and Maouche 1989).

Automated packing/nesting and automated assembly are closely related tasks. In the majority of assembly systems, the parts only fit together in certain ways, dependant on their shapes, and they can only be moved into their fitting positions in ways that are also dependent on their shape. This shape-dependent part-fitting is a key feature in any assembly application (Malcolm and Smithers 1990). A primary source of difficulty in automated assembly is the uncertainty in the relative position of the parts being assembled. This uncertainty can be significantly reduced by the use of vision and tactile sensors.

Hall, Slutzky and Shell (1989) give a good overview of the application of part nesting in intelligent robotic packaging and processing systems, as well as outlining robotic game playing systems and actual solutions to a number of industrial problems. A system for the automated palletizing of randomly arriving parcels, has been developed by the authors. In the development of such industrial systems, the authors have taken account of the some of the various systems issues involved in such a design. In such a system, heuristics are necessary to deal with the interlocking and intermeshing of the boxes on the pallets as well as dealing with the toxicity and crushability of the boxes contents. Other areas of related research include the work of Jain and Donath (1989), in which they discuss the development and implementation of a knowledge based system for three-dimensional automated assembly tasks, under robotic control.

Hoffman (1989) examines eliminating the requirement to hand tune specific assembly tasks in an automation environment. Hitakawa (1988) outlines the development of a SONY flexible automatic assembly system suitable for small-quantity batch production. Philip Chen (1991) tackles the problem of trying to find all the feasible assembly sequences for a set of n parts that construct a mechanical object. Kak *et al* (1986) outline the development of a knowledge based robotic assembly cell. This cell is used in the sensory guided part mating of three-dimensional objects. Ayache and Faugeras's (1986) paper discusses the development of a robotic system under machine vision control. This system carries out the automated picking and placement of partially overlapping industrial parts. The system is based on the generation and recursive evaluation of hypotheses for object recognition.

When faced with a specific application requirement, it is always well worthwhile analysing the problem from a systems engineering perspective. By adopting a systems approach, the maximum use is made of problem-specific "contextual" information, derived, for example, from the nature of the product being handled, the process used to manufacture it and the special features of the manufacturing environment. By doing so, the complexity of the application is hopefully reduced. For example, it may be found that, by mechanically restricting the orientation and the order of arrival of objects considered by the packing system, the problem can be simplified. In fact, by taking heed of such constraints, in a practical packing application, the procedure might well reduce to a standard, well-trying technique. It is our belief that in packing, as happens so often elsewhere, systems considerations are always worth investigating. Table 1 summarizes some of the practical considerations found in the industrial implementation of automated packing systems, that if considered within a systems engineering framework, can often reduce the complexity of a given application.

Although the application of robotics and vision to parts assembly has great potential (Harrington and Sackett 1987) and will strongly influence the competitiveness of the European community, it is currently lacking in industry. This has been recognized by the European community through its funding of major projects such as ESPRIT, BRITE and more specifically the EUREKA projects that fall under the umbrella term FAMOS³. The FAMOS-EUREKA projects have targeted one of the

³ A German acronym for Flexible Automated Assembly Systems.

weakest points in Europe's manufacturing industries, with the objective of reversing the decline of more than two decades. Its aim is to create automated assembly systems which are flexible enough to enable manufactures to change product lines when needed and to produce small batches of products efficiently. These projects include participants from a wide range of European industries and universities. Table 2 contains a brief outline of some of the FAMOS-EUREKA⁴ projects (EUREKA 1989).

PACKING CONSIDERATIONS

- Packing density.
- Object fragility.
- Object rigidity.
- Surface finish (Objects with a smooth finish will be easier to slide/push).
- Object Clampability.
- Ease of stacking.
- Packing stability.
- Load bearing ability and the effect of dynamic forces.
- Interaction between the packaging material and its contents.
- Electromagnetic radiation.
- Possibility of contamination of adjacent goods.
- Importance/value of the article.
- Protection from pilferaging.
- Need to provide ullage or vacuity.
- Probable number and nature of loadings and unloadings.
- Minimization of robot arm movement during loading and unloading (Dowland 1990).
- Material handling facilities at the final destination.
- Climatic considerations.
- Air circulation.
- Packing for aesthetic appeal (do all products need to be visible ?).
- Design and functionality of the handling equipment (e.g. robot gripper).

⁴ UPAC is an EUREKA project in this area, although it does not fall under the FAMOS umbrella.

Table 1. Practical considerations involved in industrial packing applications.

4. PACKING OF TEMPLATES ON LEATHER HIDES.

The purpose of the system outlined in this in this section, is to automatically arrange shape templates on an arbitrary shaped non-homogenous leather hide in an efficient manner, so as to minimize the leather waste. The importance of good packing⁵ procedures in the leather industry is obvious, since the raw material is both expensive and non-recyclable (in general, any waste material produced in the cutting of leather hides is sold to external companies who deal in small leather goods).

Title	Summary	Applications/Markets
JASKA EU 327	Investigating how to produce a low number of products with a high number of variants.	Mechanical components. Gearbox assembly.
ALCUT EU 335	Automatic leather cutting system.	Automotive. Upholstery. Shoe and leather industry.
UPAC EU 4	Real-time control of adaptive garment manufacturing.	Quality garment manufacturing.
INFACT EU 321	Integrated flexible assembly cell technology.	Electro-mechanical components.

Table 2. FAMOS-EUREKA projects.

In the current generation of leather cutting systems the hides pass underneath a bank of line scan cameras, which scans in a two-dimensional image of the hide to be cut. The region of the hide to be cut is then determined by the placing of shape

⁵ Recall that depletion is effectively the process of packing "holes" into a space.

templates on the scanned hide. This process is done interactively by trained operators at CAD workstations. The size and shape of these templates are application dependent. The specific application addressed (Figures 2 and 3), concerns the placing of shape templates on the leather hide, for the upholstery of high quality leather chairs and car seats. Once the position of the shape templates has been finalized, they are automatically cut from the hide by water jets (Dulio 1990; 1992).

The CAD operators have only a short period in which they can place the shape templates on the hide, that is the time between when the hide is imaged by the vision system and when the hide has progressed underneath the cutting station (in leather upholstery the operators will have to deal with 35-40 different shapes over a global surface of 55 to 60 ft² (Dulio 1992)). To this end, interactive CAD systems have been developed to aid the operator in maximising (a) the speed at which the shape templates are placed on the hide, and (b) the number of shape templates cut from a given hide. The interactive functions allow operations such as bumping, sliding and automatic repetition of shapes, quality matching, grouping and area filling.

The main packing strategy used by such operators involve the packing of the larger template pieces on the outer edges of the hide, and progressively moving in towards the centre with the remaining shapes. Some of these systems also include some semi-automation in the CAD layout. One such prototype CAD system has been developed at Dublin City University, see Figure 1.

In conjunction with a CAD company that develops such systems, we have been looking at automating this template layout process completely. That is, once the hide image is captured, a sequence of procedures will automatically layout the templates in an *efficient* manner, prior to their cutting. Figure 2 shows the automatic placement of the template pieces on a leather hide.

4.1 Packing of defective hides.

Any practical automated packing system for use in such industries as leather or timber processing must be able to pack "objects" into a scene which may contain defective regions. In a natural hide (the system is also used in the cutting of synthetic leather) there are a number of regions that cannot be used. These consist mainly of the spine and corner regions. Currently, the hide is marked using chalk or removable inks, to indicate the stress directions and aid in defect and quality recognition during the hide scanning process. Our packer can readily accommodate defects like these; we simply define the initial scene to contain a number of holes. Figure 3 shows the packing of leather templates onto such a hide. The small blob-like regions indicate the defective areas of the hide. These defective regions are not to be included in the leather pieces to be cut.

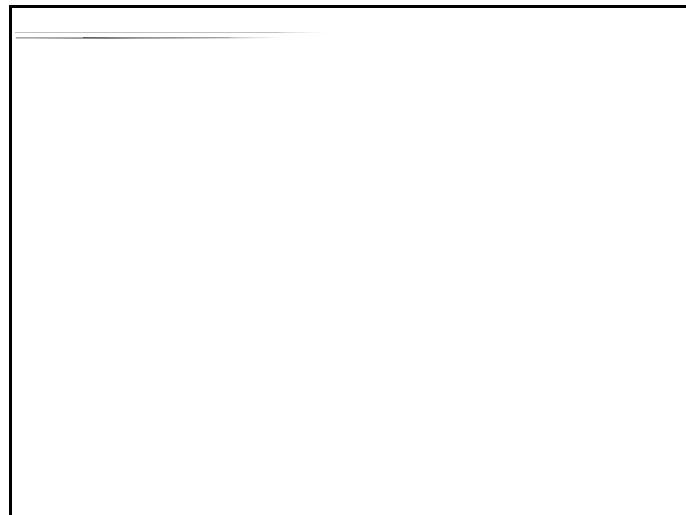


Figure 1. CAD workstation screen for the manual placement of shape templates on a leather hide.

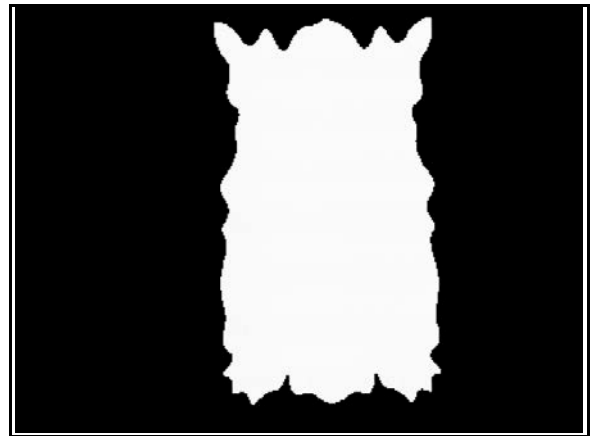
4.2 Additional points on packing in the leather industry.

The design of packing systems for the automated cutting of leather hides is made easier by the fact that leather, like fabric, wood, marble and many other natural materials has a pronounced grain or stress direction. This means that quite often only two orientations of a given leather component are permissible, a fact which can greatly enhance the speed of the packing procedure. Again, our packer can easily take this type of application constraint into account.

Packing leather component templates onto a hide is not quite as simple as we have suggested, because the leather is not uniform in its thickness and suppleness. When making shoes, for example, the components which will make up the soft leather uppers are cut from the belly of the hide, while the tougher, more rigid sole is take from the back. Adding heuristic rules to assist packing under these constraints is not difficult but has not yet been accomplished.

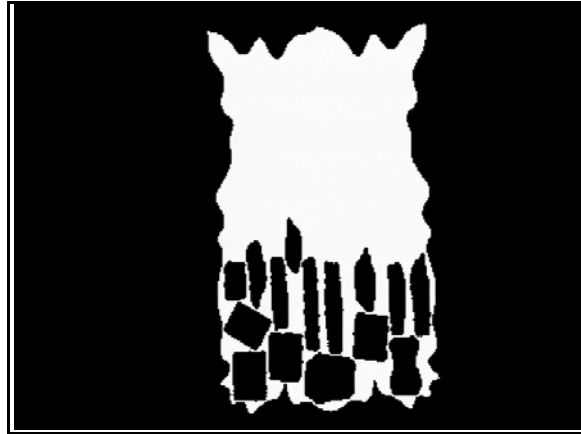
A further complication in the automated packing of the leather hides consists of the number of grades (quality levels) of leather that may exist on a given hide. Each region of a shape, depending on its importance and visibility must respect a *quality matching criteria*. The hide is subdivided into several areas of constant average quality. The shapes are also given a well defined quality, therefore each single shape, or part of a shape, can only be positioned on a portion of the hide with the **same or higher quality level** (Dulio 1990). For example, in cutting of shape templates for the manufacture of high quality leather furniture, there may be up to 40 grades of leather. Whereas in the application discussed above (high quality car seat covers) there are 5 grades of material.

One objective of such a layout system is to keep high quality parts of the hide for those components of the objects which are most visible and to try and utilize lower quality regions for non-visible parts. So in the cutting of a car seat arm rest leather component some of the leather will not be exposed to the driver, and as such it can be of a lower leather grade. This also influences the speed of the cutting operation, since lower quality parts are cut less precisely and at a higher speed. So not only do the leather pieces have to be packed to minimise waste, but the template grades must placed on the hide to suit the available grades of leather on a given hide (see Figure 4). This has not yet been implemented on the system.



(a)

(b)



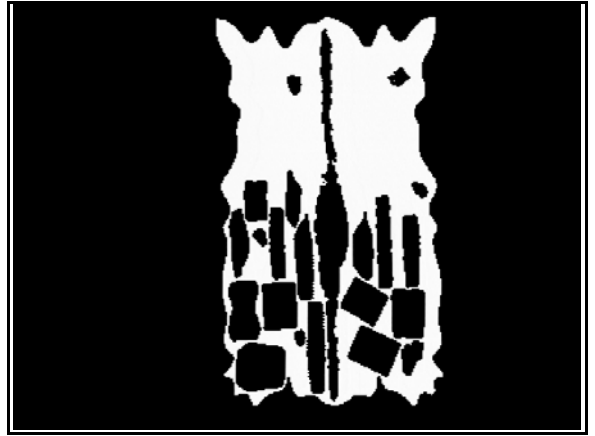
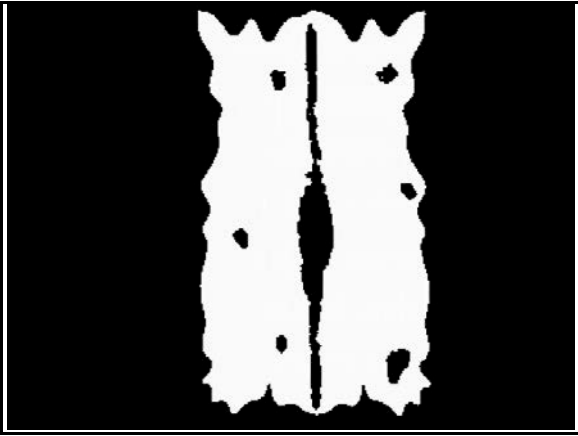
(c)

Figure 2. Automatic placement of car seat template pieces, shown in (a), on to a leather hide, (b). The resultant packed⁶ image is shown in (c).

5. CONCLUSION

Researchers in the field of production engineering, operations research, systems engineering and automation have contributed towards the large body of material used in the design of automated packing systems. This paper reviews a selection of these applications, and discusses the issues involved in the automated packing of irregular shapes. The review places its emphasis on the industrial applications of such systems, and as such highlights the systems integration issues involved in these applications. An outline of one such industrial application, the automated placement of shape templates on to leather hides, is also discussed. The problems involved in the full automation of such a procedure, are also considered.

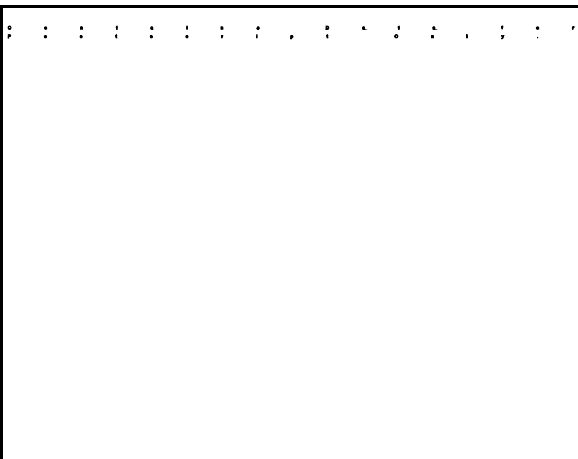
⁶ A low resolution array CCD camera has been used, in the prototype system, to capture the images in Figures 2 and 3, resulting in quantization errors on rotation of the shape pieces. The actual system will require high resolution line scan camera to build up the hide image. The packed shapes have also been spaced out (Whelan and Batchelor 1991) to clarify the resultant images.



(a)

(b)

Figure 3. Automatic placement of car seat template pieces, shown in Figure 2(a), on to a defective leather hide, (a). The black blobs in this hide, (a), indicate defective regions. The resultant packed image is shown in (b).



(a)

(b)

Figure 4. Packing of shape templates accounting for variations in leather grades. (a) Contains a leather hide with 3 leather grades (the best grade indicated by grade 1). The shape template to be packed is shown at the top of the diagram and consists of two leather grades (1 & 2). When accounting for the grade variations we must ensure that a template piece is placed over a region in the hide that is the *same or higher grade but never of a lower grade*. (b) Shows two possible packing locations for the template piece. In case A the pieces is packed according to the placement considerations described. In case B the placement cannot be allowed, since the grade on the hide is lower than the grade required by the template piece.

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